STANDARD DEVIATION

The **standard deviation** of a series of measurements which includes at least 6 independent trials may be defined as follows. If we let $\mathbf{x}_m$ be a measured value, **N** be the number of measurements, <**x**> be the average or **mean** of all the measurements, then **d** is the **deviation** of a value from the average:

$$\mathbf{d} = \mathbf{x}_m - \langle \mathbf{x} \rangle$$

and the standard deviation, **s**, is defined by:

$$s = \sqrt{\frac{\sum d^2}{(N - 1)}}$$

where $\Sigma \mathbf{d}^2$ means "sum of all the values of $\mathbf{d}^2$."

The **value of the measurement** should include some indication of the precision of the measurement. The standard deviation is used for this purpose if a large number of measurements of the same quantity is subject to random errors only. We can understand the meaning of **s** if we plot on the y-axis the number of times a given value of $\mathbf{x}_m$ is obtained, against the values, $\mathbf{x}_m$, on the x-axis. The "normal distribution curve" is bell-shaped, with the most frequent value being the average value, <**x**>.

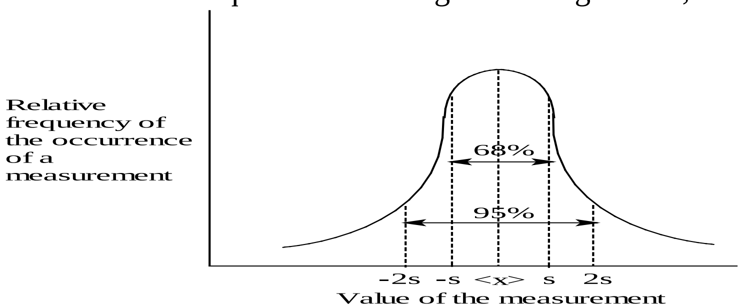

Figure 3: Distribution of Values of a Measurement

Most of the measurements give values near <**x**>. In fact, 68% of the measurements fall within the standard deviation **s** of <**x**> (see graph). 95% of the measured values are found within 2**s** of <**x**>. We call the value of 2**s** the uncertainty of the measurement, **u**. Then, if we report our value of the measurement as <**x**>**±u**, we are saying that <**x**> is the most probable value and 95% of the measured values fall within this range. The next example shows how the standard deviation can be used to evaluate the data.

Example 1. Weight of a test tube on 10 different balances

Balance Number	Weight (g)=x_m	d=x_m-<x>	d^2
1	24.29	0.00	0.0000
2	24.26	-0.03	0.0009
3	24.17	-0.12	0.0144
4	24.31	0.02	0.0004
5	24.28	-0.01	0.0001
6	24.19	-0.10	0.0100
7	24.33	0.04	0.0016
8	24.50	0.21	0.0441
9	24.30	0.01	0.0001
10	24.23	-0.06	0.0036
	Σx_m=242.86		Σd^2=0.0752

$$\langle x \rangle = 242.86/10 = 24.29\text{ g and } s = \sqrt{(0.0752/9)} = 0.0917, \text{range} = \langle x \rangle \pm 2s = 24.29 \pm 0.18\,g$$

or, the test tube weighs between 24.11 and 24.47 g, with 95% certainty.

Now each of the values of $\mathbf{x}_m$ are checked against the range. We observe that the weight from balance 8 is outside the range; we should discard it as unreliable and recalculate <**x**>, **d**, $\mathbf{d}^2$ and **s**.